



Jugowice, 11<sup>th</sup>–15<sup>th</sup> September 2017

## ANALYSIS OF A GENE EXPRESSION MODEL

Agnieszka Bartłomiejczyk<sup>1</sup>, Marek Bodnar<sup>2</sup>

<sup>1</sup>Faculty of Technical Physics and Applied Mathematics, Gdańsk University of Technology,  
ul. G. Narutowicza 11/12, 80-233 Gdańsk

<sup>2</sup>Institute of Applied Mathematics and Mechanics, University of Warsaw,  
ul. Banacha 2, 02-097 Warsaw

<sup>1</sup>agnes@mif.pg.gda.pl, <sup>2</sup>mbodnar@mimuw.edu.pl

### ABSTRACT

We study a mathematical model of gene transcription and protein synthesis with negative feedback. We consider a system of equations taking into account the number of active binding sites, the way in which dimers bind to DNA and time delay in translation process. For a simplified model that consist of three ordinary differential equations with time delay we derive conditions for stability of the positive steady state and for the existence of the Hopf bifurcation. We compare obtained results with classical model proposed by Monk in 2003.

### ACKNOWLEDGEMENTS

This work was supported by National Science Centre, Poland, project no. 2015/17/B/ST1/00693.

### REFERENCES

- [1] M. Barrio, K. Burrage, A. Leier, and T. Tian: *Oscillatory Regulation of Hes1: Discrete Stochastic Delay Modelling and Simulation*, Plos Comput. Biol. **2** (2006), 1017–1030.
- [2] A. Bartłomiejczyk and M. Bodnar: *Modelling gene expression of a self-regulating protein*, Proceedings of the Twentieth National Conference on Applications of Mathematics in Biology and Medicine, Łochów 23-27 September 2014, pp. 9–14.
- [3] A. Bartłomiejczyk, M. Bodnar, and M.J. Piotrowska: *Analysis of the p53 protein gene expression model*, Proceedings of the Twenty-First National Conference on Applications of Mathematics in Biology and Medicine, Regietów 22-26 September 2015, pp. 21–26.
- [4] S. Bernard, B. Cajavec, L. Pujon-Menjouet, M. C. Mackey, and Herzel H.: *Modelling transcriptional feedback loops: the role of Gro/TLE1 in Hes1 oscillations*, Phil. Trans. R. Soc. A **364** (2006), 1155–1170.
- [5] M. Bodnar: *Modele reakcji biochemicznych z opóźnionym argumentem: nieujemność rozwiązań i stabilność oscylacji*, Metody matematyczne w zastosowaniach (A. Bartłomiejczyk, ed.), CZM PG, 2014.
- [6] M. Bodnar and A. Bartłomiejczyk: *Stability of delay induced oscillations in gene expression of Hes1 protein model*, Non. Anal. RWA **13** (2012), 2227–2239.
- [7] K. L. Cooke and P. van den Driessche: *On zeroes of some transcendental equations*, Funkcj. Ekvacioj **29** (1986), 77–90.
- [8] H. Hirata, S. Yoshiura, T. Ohtsuka, Y. Bessho, T. Harada, K. Yoshikawa, and R. Kageyama: *Oscillatory expression of the bHLH factor Hes1 regulated by a negative feedback loop*, Science **298** (2002), 840–843.
- [9] M. Jensen, K. Sneppen, and G. Tiana: *Sustained oscillations and time delays in gene expression of protein Hes1*, FEBS Lett. **541** (2003), 176–177.
- [10] N. A. Monk: *Oscillatory expression of Hes1, p53, and NF- $\kappa$ B driven by transcriptional time delays*, Curr. Biol. **13** (2003), 1409–1413.

- [11] K. Takebayashi, Y. Sasai, Y. Sakai, T. Watanabe, S. Nakanishi, and R. Kageyama: *Structure, Chromosomal Locus, and Promoter Analysis of the Gene Encoding the Mouse Helix-Loop-Helix Factor HES-1*, J. Biol. Chem. **269** (1994), 5150–5156.
- [12] S. Zeiser, H.V. Liebscher, H. Tiedemann, I. Rubio-Aliaga, G.K. Przemeck, M.H. de Angelis, and G. Winkler: *Number of active transcription factor binding sites is essential for the Hes7 oscillator*, Theor. Biol. Med. Model. **3** (2006), 11–16.

