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INFLUENCE OF DISTRIBUTED DELAYS ON THE DYNAMICS OF A CANCER–IMMUNE SYSTEM INTERACTIONS MODEL

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ABSTRACT

In [2] a simple ODE model describing interactions between tumour and immune system was proposed. Latter on in [3] that model was generalised by considering a general form of the stimulus function f and in addition a discrete time delay (τ) representing time lag in the immune system response to presence of antigens was incorporated into the model. We further generalise the model from [3] by considering instead of discrete delay a distributed one.

We analyse basic mathematical properties of generalised model and discuss the existence of steady states and theirs stability depending on the forms of considered probability densities (ones with compact support (triangular and uniform) and that without compact support (Erlang) including possible shifts). We show that for unstable (for ODE model) steady states an introduction of the distributed delay will not change theirs stability – that holds for a whole class of distributed delays. We also show that the stability of the tumour free steady state does not depend on delay.

To evaluate the model with experimental data for mice B-cell lymphoma, [5], we adjust the idea of linear chain trick [4] and derive numerical procedures allowing to solve the systems with distributed delays using standard algorithms for ODEs or differential equations with discrete delays. Our fitting results show a good agreement of model solutions with the experimental data with mean square errors at the same comparable level for all considered probability densities.

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REFERENCES

[1] M.J. Piotrowska and M. Bodnar: *Influence of distributed delays on the dynamics of a generalized immune system cancerous cells interactions model*, Commun. Nonlinear. Sci. Numer. Simul., (accepted).

[2] U. Foryś, J. Waniewski, and P. Zhivkov: *Anti-tumor immunity and tumor anti-immunuty in a mathematical model of tumor immunotherapy*, J. Biol. Sys. **14(1)** (2006), 13–30.

[3] M.J. Piotrowska: *An immune system—tumour interactions model with discrete time delay: Model analysis and validation*, Commun. Nonlinear. Sci. Numer. Simul. **34** (2016), 185–198.

[4] Y. Kuang: *Delay differential equations with applications in population dynamics*, 1993.

[5] J.W Uhr, T. Tucker, R.D May, H. Siu, and E.S Vitetta: *Cancer dormancy: Studies of the murine BCL_1 lymphoma*, Cancer Res. (Suppl.) **51** (1991), 5045s–5053s.