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MICROSCOPIC APPROXIMATIONS OF MACROSCOPIC MODELS FOR VEHICULAR AND PEDESTRIAN FLOWS

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ABSTRACT

In this talk we present recent results on the deterministic particle approximation of nonlinear conservation laws. The unique entropy solution to the scalar conservation law

$$\rho_t + \big[\rho\, v(\rho)\big]_x = 0 \tag{1}$$

with a given initial datum in $\mathbf{L}^\infty$ and with strictly monotone v was rigorously approximated in [1, 3] by the discrete density constructed from the follow-the-leader particle system

$$\dot{x}_i(t) = v\left(\frac{m}{x_{i+1}(t) - x_i(t)}\right). \tag{2}$$

Said result is based on a discrete version of the classical Oleinik one-sided jump condition for $\mathbf{L}^\infty$ initial data and on a $\mathbf{BV}$ contraction estimate for $\mathbf{BV}$ initial data. The former requires some additional conditions on v. The IBVP for (1) has been considered in [5]. The results in [1] have been extended to the 2×2 system of conservation laws describing the Aw-Rascle-Zhang (ARZ) model for vehicular traffic in [2], where a similar $\mathbf{BV}$ contraction estimate has been proven, based on the interpretation of the ARZ model as a multi-population model. Finally, we present an extension of these techniques to the one dimensional version of the Hughes model for pedestrians

$$\rho_t - \left[\rho\, v(\rho)\, \frac{\phi_x}{|\phi_x|}\right]_x = 0\,, \qquad\qquad |\phi_x| = c(\rho)\,, \tag{3}$$

on a bounded interval with Dirichlet boundary conditions. In [4] we prove the rigorous convergence of a suitable adaptation of the particle scheme (2) to the unique entropy solution to the IBVP for (3).

REFERENCES

[1] Di Francesco M. and Rosini M. D.: *Rigorous derivation of nonlinear scalar conservation laws from follow-the-leader type models via many particle limit*, Archive for rational mechanics and analysis **217** (2015), 831-871.

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[3] ———: *Deterministic particle approximation of scalar conservation laws* (submitted).

[4] Di Francesco M. and Fagioli S. and Rosini M. D. and Russo G.: *Deterministic particle approximation of the Hughes model in one space dimension* (submitted).

[5] ———: *Microscopic approximations of macroscopic models for vehicular and pedestrian flows* (submitted).