



Sandomierz, 5th–9th September 2016

DELAYS DO NOT CAUSE OSCILLATIONS IN A CORRECTED MODEL OF HUMORAL MEDIATED IMMUNE RESPONSE

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ABSTRACT

we focus on modelling immune system - cancer cells interactions with time delays included into the model. The first model we consider, is the Feyissa and Banerjee model of humoral immune response [2], for which we show that the non-negativity property is not preserved. This is the reason we propose a modification which consists of five ordinary differential equations with time delay. Under the assumption that the tumour is non-immunogenic can be simplified into the system of three equations without delays, that after rescaling reads

$$\begin{aligned}\dot{x}(t) &= k \left(\alpha - \eta x(t) - x(t) y(t) \right), \\ \dot{y}(t) &= \gamma y(t) \left(1 - y(t) - z(t) \right) - x(t) y(t), \\ \dot{z}(t) &= x(t) y(t) - z(t),\end{aligned}$$

where x , y , z are rescaled concentrations of antibodies, cancerous cells, and damaged cancerous cells respectively. For the simplified model we have made mathematical analysis, including asymptotic stability, as well as numerical analysis, showing wide spectrum of possible model dynamics, including globally stable steady state, bistability and oscillatory behaviour. The dynamics of the simplified model is numerically compared with the dynamics of the full model with time delays for the parameter values from the literature. It occurs that if tumour immunogenicity is low time delays have no significant influence on the full system and both systems behave similarly.

ACKNOWLEDGEMENTS

The work on this paper was supported by the Polish Ministry of Science and Higher Education, within the Iuventus Plus Grant: “Mathematical modelling of neoplastic processes” grant No. IP2011 041971.

REFERENCES

- [1] M. Bodnar and U. Foryś: *Delays do not cause oscillations in a corrected model of humoral mediated immune response*, Appl. Math. Comput. **289** (2016), 7–21.
- [2] S. Feyissa and S. Banerjee: *Delay-induced oscillatory dynamics in humoral mediated immune response with two time delays*, Nonlinear Analysis: Real World Applications **14** (2013), 35–52.