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FORECASTING MALARIA INCIDENCE IN RWANDA USING NEURAL CONTROLLED DIFFERENTIAL EQUATIONS INTEGRATED WITH OPTIMAL CONTROL TOOLS

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ABSTRACT

Malaria remains a major public-health concern in Rwanda. After an 85% reduction between 2019 and 2023, the country recorded a resurgence to about 518,000 infections in the first nine months of 2024, while Africa continues to bear roughly 95% of an estimated 282 million cases reported worldwide. Designing time-dependent intervention schedules that balance epidemiological impact and operational cost is a nonlinear, vector-mediated optimal-control problem for which classical value-function and Pontryagin-type approaches are difficult to apply, because the value function determined by the running cost cannot be expected to be smooth for nonlinear vector–host systems.

We formulate malaria control in Rwanda as a continuous-time optimal control problem governed by a SEIRS–SEI compartmental model with state $Z(t) = (S, E, I, R, I_m)^\top \in \mathbb{R}^5$ – the susceptible, exposed, infectious and recovered human compartments together with the infectious-mosquito proportion – and two time-dependent controls $u(t) = (u_p(t), u_k(t)) \in \mathcal{U} = [0, 1]^2$: u_p aggregates vector control (long-lasting insecticidal nets, indoor residual spraying, larval source management), modifying the biting and mosquito-mortality rates, while u_k represents treatment effort acting on the human recovery rate. Given the controlled dynamics $\dot{Z}(t) = F(Z(t), u(t))$ with $Z(0) = Z_0$, the primal optimal-control problem is

$$\min_{u(\cdot) \in \mathcal{U}} J(u) = \int_0^T \left[A_1 I(t) + A_2 I_m(t) + \frac{1}{2} A_3 \|u(t)\|^2 \right] dt,$$

which penalises human infectious prevalence I , infectious-mosquito proportion I_m and intervention effort, with positive weights $A_1, A_2, A_3 > 0$.

To solve this problem we adopt a *dual dynamic programming* (DDP) framework. Rather than work with a value function – which need not be smooth for the nonlinear five-dimensional system – we work with an auxiliary pair (q, Q) defined on a dual parameter space $P = B(0, \rho) \subset \mathbb{R}^5$ satisfying the *dual dynamic equality*

$$\frac{\partial Q}{\partial t}(t, p) + \inf_{u \in [0, 1]^2} [p^\top F(p \cdot Q, u) + L(p \cdot Q, u)] = q(t), \quad Q(0, p) = c \neq 0,$$

the Hamilton–Jacobi analogue of the framework on P . The dual arc $p(\cdot)$ is determined purely on P by a dual trajectory equation with terminal condition $p(T) = 0$, and the recovered state $Z_{(q, Q)}(t; u) := p(t) \cdot Q(t, p(t))$ is reconstructed from p ; the requirement $Z_{(q, Q)} = Z(\cdot; u)$ is a consistency condition on (q, Q) , not a defining equation for p .

On this foundation we establish two theoretical results. An *Optimality Verification Theorem* certifies that any candidate control satisfying the verification equality $\partial_t Q + p^\top F(Z_{(q, Q)}, u) +$

$L(Z_{(q,Q)}, u) = q$ is optimal for the primal problem, with a proof that uses neither Pontryagin’s maximum principle nor an adjoint boundary-value problem. The associated residual $r(t) := q(t) - \partial_t Q - p^\top F(Z_{(q,Q)}, u) - L(Z_{(q,Q)}, u)$ supplies a computable ε -optimality certificate, and a complementary theorem provides sufficient conditions, expressed through the effective reproduction number $\mathcal{R}_{\text{eff}}$, under which prevalence remains below a chosen threshold during an initial window.

These results are assembled into a three-stage procedure. *Stage 0* constructs the auxiliary pair (q, Q) by solving the dual dynamic equality as a parametrised scalar ODE in p : with $q \equiv 0$ and the dual variable p ranging continuously over P , each fixed p yields an initial-value problem whose closed-form pointwise minimiser $u^*(t, p) = \Pi_{[0,1]^2}(-A_3^{-1}b)$ – with $b = (\beta Q^2 p_1 p_5 (p_1 - p_2), Q p_3 (p_4 - p_3))$ – delivers $Q(t, p)$ on $[0, T] \times P$ by fourth-order Runge–Kutta integration, without any prescribed (e.g. affine) shape for Q . *Stage 1* generates a candidate intervention schedule by projected back-tracking gradient descent on $\mathcal{U} = [0, 1]^2$ from a constant initial guess $u^{(0)} \equiv 0.30$, simulating the primal state from $\dot{Z} = F(Z, u)$ and updating $u^{(n+1)} = \Pi_{\mathcal{U}}(u^{(n)} - \alpha^{(n)} \nabla_u J(u^{(n)}))$. *Stage 2* propagates the dual arc $p(\cdot)$ backward on P from $p(T) = 0$ via the dual trajectory equation, recovers the state through $Z_{(q,Q)}(t; u) = p(t) \cdot Q(t, p(t))$, verifies the matching identity $Z_{(q,Q)} = Z(\cdot; u)$ and evaluates the residual $r(t)$, realised as the projected-gradient variational-inequality residual $r(t) = u^*(t) - \Pi_{[0,1]^2}(u^*(t) - \nabla_u J[u^*](t))$, whose supremum norm $\varepsilon = \sup_{t \in [0, T]} |r(t)|$ is the ε -optimality certificate of the verification theorem.

This procedure is applied to a Rwanda-specific parameter set drawn from the malaria-modelling literature: a planning horizon of $T = 180$ days (one transmission season), objective weights $(A_1, A_2, A_3) = (100, 10, 1)$ that prioritise reduction of human infectious prevalence while still penalising intervention effort, and SEIRS–SEI parameters covering human demography ($\mu = 4.09 \times 10^{-5}$), incubation and recovery ($\sigma = 0.065, \gamma = 0.05$), loss of immunity ($\omega = 0.00274$), and the vector chain ($a = 0.30, \beta = 0.12, c = 0.45, \mu_m = 0.0714$, all in day^{-1}). The controlled dynamics are integrated by a fourth-order Runge–Kutta scheme with step 0.25 day, the control $u(\cdot) = (u_p, u_k)$ is represented on a mesh of 61 nodes taking continuous values in $[0, 1]^2$, and the initial state $Z_0 = (0.80, 0.04, 0.045, 0.115, 0.09)^\top$ uses an infectious prevalence consistent with the malaria resurgence reported for Rwanda in 2024. The presentation will discuss the SEIRS–SEI formulation, the dual dynamic equality and verification theorem, the construction of (q, Q) on the continuous dual space P , and the three-stage ε -optimality algorithm applied to this Rwanda parameter set.

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