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WHEN TIME MATTERS: DELAY DIFFERENTIAL EQUATIONS IN MATHEMATICAL BIOLOGY

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ABSTRACT

Biological processes take time, and this time lag can have a profound impact on the dynamics of the system. When the duration of a biological process is comparable to the time scale of the population or biochemical dynamics, neglecting it may lead to qualitatively incorrect conclusions. A natural way to incorporate such effects into mathematical models is through time delays.

In this presentation, I will discuss delay differential equations and their applications in mathematical biology. I will briefly introduce the mathematical foundations of the theory and explain how the presence of a delay changes the phase space from the finite-dimensional space $\mathbb{R}^n$ to an infinite-dimensional Banach space. This fundamental difference means that some familiar intuition from ordinary differential equations no longer applies.

The main focus of the presentation will be the impact of time delays on the stability and dynamics of stationary solutions. A delay can destabilize an otherwise stable equilibrium and lead to the emergence of persistent oscillations. On the other hand, delays may also have a stabilizing effect, and multiple stability switches can occur as the delay varies. I will present mathematical techniques for determining the stability of equilibria, locating critical delay values, and identifying transitions between stable and unstable regimes.

I will also discuss the possibility of periodic solutions and the role of delay-induced oscillations in biological models. The theoretical ideas will be illustrated with examples motivated by biological processes, showing how incorporating delays can lead to qualitatively new dynamical behavior.

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