Wikno, 16th–20th September 2025

AROUND LOTKA-VOLTERRA MODELS WITH DIFFUSION AND TAXIS

Dariusz Wrzosek

Institute of Applied Mathematics and Mechanics , University of Warsaw
Banacha 2, 02-097 Warsaw
darekw@mimuw.edu.pl

ABSTRACT

We study a prey-predator system and a competition system, both with Lotka-Volterra reaction terms, assuming, in addition to the diffusion movement of both species, an avoidance strategy of one of them modeled as repulsive taxis.

$$N_t = D_n\Delta N + \nabla\cdot(\chi N\nabla P) + \alpha_1 N - \alpha_2 N^2 - \alpha_3 NP\,,$$
$$P_t = D_p\Delta P + \beta_1 P - \beta_2 P^2 + \beta_3 NP$$

in $\Omega\times(0\,,\infty)$ where N and P denote the densities of prey and predator or the densities of two competitors. It is assumed that the set $\Omega\subset R^n$ is a region with a smooth boundary. The system is suplemented by initial conditions and Neumann, no-flux, boundary conditions. The coefficients $D_n\,, D_p$ are diffusion constants, $\chi > 0$ is a taxis sensitivity coefficient. The taxis term describes the movement against the predator or competitor density in order to reduce the frequency of encounters.

The aforementioned model describes the mechanism of so called direct taxis. The taxis is called direct if the animals are guided by the density gradient of another population or indirect if they are guided by the density of a chemical secreted by individuals of another population. It is interesting to understand the relation between both models of taxis and in paper [3] we consider the asymptotic transition from the model with indirect taxis to the model with direct taxis as a fast reaction limit.

The prey-predator model and the competition model, can be considered from the perspective of the itraguild predation and then the sign of the parameter β_3 changes from $\beta_3 < 0$ (competition) to predation, $\beta_3 > 0$, depending on the carrying capacity of the common food resources for both species. The problem of blow-up prevention for the models turns out to be challenging for space dimension $n > 1$ and we present partial results from [2]. Mechanism of pattern formation will be described for the models and illustrated by the results of numerical simulations (c.f. [1, 2]).

REFERENCES

[1] P. Mishra and D Wrzosek: *Schoener-Polis-Holt's model of the intraguild predation with predator taxis and repulsive chemotaxis*, Discrete Contin. Dyn. Syst. Ser. B **29** (2024), 4698-4726.

[2] _____: *Global existence for Lotka-Volterra models of predation and competition with diffusion and predator/competitor taxis in the spatial dimension n = 2, 3.*, submitted.

[3] J.I Tello and D Wrzosek: *From indirect to direct taxis by fast reaction limit*, submitted.