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A HYBRID STOCHASTIC MODEL OF RETINAL ANGIOGENESIS

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ABSTRACT

Understanding the biological principles that govern blood vessel growth in the retina has important clinical implications, for the prevention of possible retinopathies, which may eventually lead to blindness. We present a mathematical model that describes vessel formation by branching diffusion of Langevin type with chemotaxis due to fields of concentrations governed by PDEs. There are two types of cells in the model, the cells undergo proliferation and can change their type. The movement of vessel tips is given by the following Langevin type stochatic equation

$$\begin{cases} d\mathbf{X}_i^2(t) = & \mathbf{V}_i^2(t)dt \\ d\mathbf{V}_i^2(t) = & [-k\,\mathbf{V}_i^2(t) + F(g, \nabla g, u, \nabla u)]dt + \sigma dW_i(t), \end{cases}$$

where u and u are concentration fields of vascular growth factor and oxygen, that evolve according to the following PDEs

$$\frac{\partial u(t,\mathbf{x})}{\partial t} = -d_u u(t,\mathbf{x}) + D_u \Delta u(t,\mathbf{x}) + \alpha_u \eta(t,\mathbf{x}, Q^{[1]}(t)),$$

$$\frac{\partial g(t,\mathbf{x})}{\partial t} = -d_g g(t,\mathbf{x}) + D_g \Delta g(t,\mathbf{x}) + S\,(u(t,\mathbf{x})) - \alpha_g g(t,\mathbf{x})\eta(t,\mathbf{x}, Q^{[1]}(t)),$$

We call our model hybrid since it includes the coupling of a fully stochastic model for the construction of a vessel network in the retina, with continuum underlying fields describing relevant factors, such as growth factors and oxygen.

The model leads to numerical simulations that somehow reproduce normal vascularization, and predict possible pathologies.

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REFERENCES

[1] V. Capasso and R. Wieczorek: *A hybrid stochastic model of retinal angiogenesis*, Mathematical Methods in the Applied Science **43** (2020), 10578–10592.