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PANGRAPHS AS MODELS OF HIGHER-ORDER INTERACTIONS

Alesandra Puchalska[1]

[1]Faculty of Mathematics Informatics and Mechanics, University of Warsaw,
Banacha 2, 02-097 Warsaw
[1]apuchalska@mimuw.edu.pl

ABSTRACT

Graphs have long been effective models for pairwise interactions, providing valuable insights into the fundamental properties of trophic and mutualistic networks. However, it is now increasingly recognized that non-pairwise (higher-order) interactions play a crucial role in influencing the stability of underlying dynamic systems. Hypergraphs, which allow edges to connect arbitrary subsets of vertices, have been proposed as models capable of incorporating these interactions. Despite this, their application often obscures the roles of individual vertices and magnifies centrality measures.

In this work [1], we propose an extension of the hypergraph concept that more effectively captures complex higher-order interactions, including the modification of information. We introduce the concept of a pangraph, an extension of the ubergraph [3] into directed graphs. In a pangraph, edges can start and/or end at other edges, with arbitrary levels of nesting. This framework enables a more consistent representation of complex interactions. In this talk, we will explore the properties of pangraphs, their relationship to classical directed graphs through the Levi representation, and propose centrality measures tailored to these structures. We will also discuss potential methods for extending the pangraph concept to the dynamics, mainly transport equation, on metric graphs.

Finally, we highlight two compelling applications of pangraphs. The first applies the pangraph framework to structure the theory of Petri nets with catalysts. The second revisits a hypothesis about the significance of interaction modifications, as presented in [2]. Using a real-world coffee ecosystem database, we show that the results in [2] can be interpreted as an amplification of centrality measures in the hypergraph model.

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REFERENCES

[1] M. Iskrzyński, A. Puchalska, A. Grzelik, and G. Mütlu: *Pangraphs as models of higher-order interactions*, 2024.

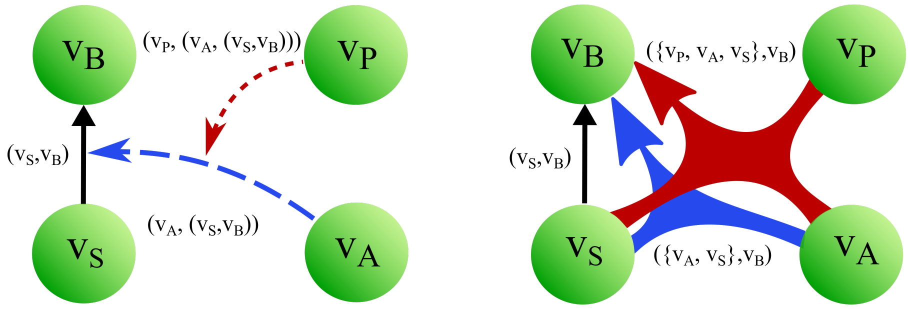

Figure 1. A 4-node subgraph of the coffee agroecosystem model from [1] is presented here in two representations: as a pangraph (on the left) and as a hypergraph (on the right). In this diagram, the nodes v_P, v_A, v_S and v_B correspond to Phorid, Azteca, Scale, and Beetle, respectively, as described in [2]. The colours in the pangraph indicate the depth of objects within it: fundamental vertices (with zero depth) are shown in green, one-depth edges are black, two-depth panedges are blue, and three-depth panedges are red.

[2] A. J. Golubski, E. E. Westlund, J. Vandermeer, and M. Pascual: *Ecological networks over the edge: Hypergraph trait-mediated indirect interaction (tmii) structure*, Trends in Ecology and Evolution **31** (2016), 344 – 354.
[3] C. Joslyn and K. E. Nowak: *Ubergraphs: A definition of a recursive hypergraph structure*, 2017.