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ON INSTABILITY OF PREY-PREDATOR SYSTEM

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ABSTRACT

In this paper, we consider a specific prey–predator system with convective terms in one spatial dimension, $x \in [0, L]$

$$\begin{aligned} N_t + (NV)_x &= -\alpha N n, \\ n_t + (nv)_x &= \beta N n. \end{aligned} \tag{1}$$

We assume that there exists $a \in [0, L]$ such that $v_x < 0$ on $[0, a)$ and $v_x > 0$ on $(a, L]$, so that $v(a) = 0$. For simplicity, we assume that V is constant. Under these conditions we need only one boundary condition at $x = 0$, i.e.

$$N_t(0) = N_0.$$

Assuming high typical mobility of the predator, we can separate the evolution time scales and introduce a small parameter. Roughly speaking, this means that $1/\alpha$ is large. We show that, to good approximation, the system can be reduced to a single linear partial differential equation for N and an ordinary differential equation for $J(t)$:

$$\begin{aligned} N_t + (NV)_x &= -R(x)N, \\ \frac{d}{dt}J(t) &= \lambda J(t), \end{aligned} \tag{2}$$

where $-\lambda$ is an eigenvalue of the operator

$$\mathcal{L}(t) = v(x)\frac{d}{dx} + v_x + N(t, x),$$

which depends parametrically on t.

The system (2) is much easier to analyze. In general, it has oscillatory solutions. For large β it is possible to find an analytical form of these solutions.