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MODELLING OF IMMUNE RESPONSE WITH ECOLOGICAL FACTOR

Yaroslav Bihun[1], **Oleh Ukrainets**[2]

[1,2]Department of Applied Mathematics and Information Technologies,
Yuriy Fedkovych Chernivtsi National University
str. Universytetska 28, Chernivtsi, Ukraine
[1]y.bihun@chnu.edu.ua, [2]o.ukrainets@chnu.edu.ua

ABSTRACT

In this paper, we investigate the G. Marchuk model [1] of an immune system with influence of the ecological factors $E(t)$ (air pollution, water quality, etc.), which can negatively affect the immune response to infectious diseases. We proposed an average indicator $E(t)$ of the ecological impact, which satisfies the general Hutchinson equation and is represented as follows:

$$\frac{dE(t)}{dt} = r\left(1 - \left(\frac{E(t-\Delta)}{E^*}\right)^n\right)E(t), t > 0, \tag{1}$$

where $r > 0$ denotes the linear growth rate, and $\Delta > 0$ represents the average time required to restore ecological balance, with the equilibrium level given by $E^* > 0$, $n \geq 1$.

A mathematical model of the immune response to infectious diseases was developed by G. Marchuk [1], was studied in the works of U. Forys and M. Bodnar, for example [2], and other authors. This report considers a mathematical model of the form:

$$\begin{aligned} \frac{dV}{dt} &= (\beta - \gamma F)V, \\ \frac{dC}{dt} &= \alpha\xi(m)V_\tau F_\tau - \mu_c(C - C^*) - \varepsilon_c E, \\ \frac{dF}{dt} &= \rho C - (\mu_f + \eta\gamma V)F, \quad \frac{dm}{dt} = \sigma V - \mu_m m + \varepsilon_m E, \end{aligned} \tag{2}$$

where $V(t), C(t), F(t)$ and $m(t)$ are the amounts of antigen population, cascade of plasma cells, antibodies and a generalized measure of organ damage caused by the infection accordingly, considering $0 \leq m \leq 1$, $V_\tau(t) = V(t-\tau), 0 < \tau$ – time of active immune response, coefficients of model (2) are non-negative numbers.

The existence of a non-negative solution to the system (1), (2) for $t > 0$ has been established. Conditions for the existence of stationary solutions corresponding to the healthy state and the chronic form of the disease have been determined. The local asymptotic stability of these solutions has been analyzed. Numerical simulations were performed to explore disease dynamics under varying model parameters and environmental influence.

REFERENCES

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