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WHY DO COMPUTERS LIKE LORENZ MAPS?

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ABSTRACT

Lorenz maps, initially inspired by the famous Lorenz attractor, have evolved into a rich family of one-dimensional dynamical systems that are simple to define yet capable of exhibiting highly complex behavior. These maps, including β-transformations and expanding Lorenz maps, arise naturally in diverse contexts - from symbolic dynamics to computational neuroscience. In this talk, we investigate why such maps are particularly appealing from a computational standpoint. We introduce the concepts of *numerical transitivity* and *numerical leo* (locally eventually onto), which offer practical tests to assess how thoroughly a map "explores" its domain under iteration. These notions serve as computational proxies for dynamical properties like chaos and mixing, allowing us to visualize and quantify behavior through finite-time simulations. We discuss algorithms for both properties and present illustrative examples using β-transformations, revealing surprising regions of apparent chaotic behavior even in seemingly simple systems. This exploration not only sheds light on the numerical behavior of Lorenz-type maps but also demonstrates their computational utility in modeling real-world phenomena. Finally, we show applications of our numerical methods to the analysis of the 1D Courbage-Nekorkin-Vdovin model of a single neuron (see [1–3]). This is joint work with Rudrakshala Kavya sri and Sishu Shankar Muni (School of Digital Sciences, Digital University Kerala, India).

REFERENCES

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