



Zdunya, 17<sup>th</sup>–21<sup>st</sup> September 2024

## HOSPITAL COMPETITION WITH AGE-STRUCTURED PATIENTS AND CONGESTION EFFECTS: A DIFFERENTIAL GAME APPROACH

**Michael Kuhn<sup>1</sup>, Dominika Machowska<sup>2</sup>, Andrzej Nowakowski<sup>3</sup>, Agnieszka  
Wiszniewska-Matyszkiewicz<sup>4</sup>, Stefan Wrzaczek<sup>5</sup>**

<sup>1,5</sup>International Institute for Applied Systems Analysis, Schlossplatz 1, 2361 Laxenburg, Austria;  
Wittgenstein Center (IIASA, OeAW/VID, University of Vienna), Austria,

<sup>2,4</sup>Faculty of Mathematics, Informatics and Mechanics, University of Warsaw  
ul. Banacha 2, 02-097 Warszawa,

<sup>3</sup>Faculty of Mathematics and Computer Science, University of Łódź,  
Banacha 22, Łódź 90–238, Poland

<sup>1</sup>kuhn@iiasa.ac.at, <sup>2</sup>machowska@mimuw.edu.pl,

<sup>3</sup>annowako@math.uni.lodz.pl, <sup>4</sup>agnese@mimuw.edu.pl,

<sup>5</sup>wrzaczek@iiasa.ac.at

### ABSTRACT

We explore a noncooperative game framework involving two hospitals, where treatment quality suffers under congestion. Recognizing that healthcare demand is significantly influenced by patient age, we incorporate a continuous age distribution into our model. Each hospital aims to determine the optimal treatment (age-structured) intensity that maximizes its objective: for a public hospital, this involves enhancing the cross life-expectancy as measure for the number and quality of treated patients (public hospital); for a private hospital, the goal is to maximize profits based on public payments for treatment. The resulting problem leads to the introduction of differential games with the closed-loop information structure. The paper formulates conditions for verifying whether a given strategy profile constitutes an  $\varepsilon$ -Nash equilibrium with the dual closed-loop information structure. The verification theorem is then used to develop a numerical algorithm for determining  $\varepsilon$ -Nash equilibria in a finite number of steps. The numerical simulations demonstrate how the Nash equilibrium can shift in response to varying socio-economic factors.