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ANALYTICAL AND NUMERICAL APPROACHES FOR SOLVING SINGULAR OPTIMAL CONTROL PROBLEMS: A CASE STUDY

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ABSTRACT

Solving an optimal control problem involves identifying the control structure and corresponding switching times. Unlike in the bang-bang case, switching to a singular control disrupts the control structure. The perturbation of one switching time affects subsequent singular intervals in the control, as the trajectories follow different singular arcs with varying values of singular controls. This makes finding optimal solutions extremely challenging.

The main aim of this study is to demonstrate the importance of the mathematical analysis of the model and the analytical investigation of the optimal control structure. Numerical optimisation methods can yield incorrect results due to the computational complexity of the problem. Comparing numerical results with analytical ones allows us to discard solutions that result from errors and numerical artefacts.

To illustrate the significance of the analytical approach, we conduct a case study [1]. We formulate a simple mathematical model of competition between two species, where one is sensitive to control and the other is resistant. Following the idea of [2], we define a nonlinear objective functional aimed at preventing the domination of one species over the other. Using the Pontryagin Maximum Principle, we derive the first-order optimality conditions and examine the structure of the optimal controls. Subsequently, we solve the problem numerically using three popular optimisation modelling languages: AMPL, JuMP, and Pyomo. These languages serve as interfaces for solving the optimal control problem with the non-linear optimisation algorithm Ipopt. Finally, we compare the results with those obtained using the gradient algorithm proposed in [2]. We show that in complex computational problems, a crucial element of the gradient method is the use of an appropriate gradient descent algorithm. For this purpose, we employ a learning method based on the Adaptive Moment Estimation (Adam) algorithm, which offers advanced techniques for adjusting the learning rate, enhancing the stability and speed of convergence of the algorithm.

REFERENCES

[1] M. Bodzioch: *Gradient method for solving singular optimal control problems*, Computational Science – ICCS 2024, Lecture Notes in Computer Science **14836** (2024) (to appear).

[2] P. Bajger, M. Bodzioch, and U. Foryś: *Singularity of controls in a simple model of acquired chemotherapy resistance*, Discrete and Continuous Dynamical Systems Series B **24(5)** (2019), DOI 10.3934/dcdsb.2019083, 2039–2052.