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DYNAMICAL SYSTEMS ON NETWORKS — ON THE CROSSROADS OF DISCRETE AND CONTINUOUS MATHEMATICS

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ABSTRACT

Recently there has been an interest in dynamical problems on graphs, where some evolution operators, such as transport or diffusion, act on the edges of a graph and interact through nodes. One can mention here quantum graphs, diffusion on graphs in probabilistic context, transport problems, both linear and nonlinear, or migrations. In this talk we shall focus on general linear transport problems posed on networks consisting of one dimensional domains, called edges, which are coupled through transmission conditions between an arbitrary selection of the endpoints of the edges. This allows for communication between domains which not necessarily are physically connected, and makes it possible to consider, within the same framework, not only classical transport and diffusion problems on graphs but also models such as Rotenberg type models describing mutations in dividing cells.

We address the following problems:

(1) Well-posedness of the problems;
(2) Conditions under which such generalized models have classical graph representation;
(3) Asymptotic periodicity of network transport.
(4) Asymptotic state lumping; that is, conditions under which such network problems can be approximated by appropriately constructed system of ordinary differential equations.

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