INTERSPECIES CORRELATION FOR NEUTRALLY EVOLVING TRAITS

Serik Sagitov\textsuperscript{1} and Krzysztof Bartoszek\textsuperscript{2}

\textsuperscript{1}Mathematical Sciences, Chalmers University of Technology and the University of Gothenburg G"oteborg, Sweden, \textsuperscript{2}Department of Mathematics, Uppsala University Uppsala, Sweden, \\
\textsuperscript{1}serik@chalmers.se, \textsuperscript{2}krzysztof.bartoszek@math.uu.se

ABSTRACT

A simple way to model phenotypic evolution is to assume that after splitting, the trait values of the sister species diverge as independent Brownian motions. Relying only on a prior distribution for the underlying species tree (conditioned on the number, \( n \), of extant species) we study the random vector \((X_1, \ldots, X_n)\) of the observed trait values. In this paper we derive compact formulae for the variance of the sample mean and the mean of the sample variance for the vector \((X_1, \ldots, X_n)\). The key ingredient of these formulae is the correlation coefficient between two trait values randomly chosen from \((X_1, \ldots, X_n)\). This interspecies correlation coefficient takes into account not only variation due to the random sampling of two species out of \( n \) and the stochastic nature of Brownian motion but also the uncertainty in the phylogenetic tree. The latter is modeled by a (supercritical or critical) conditioned branching process. In the critical case we modify the Aldous–Popovic model by assuming a proper prior for the time of origin.

ACKNOWLEDGEMENTS

We would like to thank Graham Jones for numerical procedures for the calculation of \( e_{n,m} \) and providing R code for this. We would also like to thank Thomas F. Hansen and anonymous reviewers for helpful suggestions on improving our work. The research of Serik Sagitov was supported by the Swedish Research Council Grant 621-2010-5623. Krzysztof Bartoszek was supported by the Centre for Theoretical Biology at the University of Gothenburg, Stiftelsen för Vetenskaplig Forskning och Utbildning i Matematik (Foundation for Scientific Research and Education in Mathematics), Knut and Alice Wallenbergs travel fund, Paul and Marie Berghaus fund, the Royal Swedish Academy of Sciences and Wilhelm and Martina Lundgrens research fund.

REFERENCES