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LOGISTIC TYPE EQUATIONS WITH DISCRETE DELAY AND QUASI-PERIODIC SUPPRESSION RATE

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ABSTRACT

In this paper we study stability of steady states of the logistic type models with time delay and non-autonomous periodic (or quasi-periodic) suppression rate. Logistic type models are often used to model a single species dynamics, including the underlying dynamics of tumour cells. Moreover, time delay is sometimes necessary to better reflect the description of a real process or to fit the model to experimental data, see e.g. [?mbuf07jbs, ?jain12aml]. On the other hand, an external influence reflected by the suppression function can describe some kinds of the tumour treatment, such as chemotherapy, radiotherapy or immunotherapy. Typically, such treatments are periodic, and therefore we are mainly interested in periodic suppression functions. However, the results for periodic functions can be easily extended for a quasi-periodic case.

More precisely, we consider the following equation

$$\dot{x}(t) = \alpha\Big(f\Big(x(t-1)\Big) - s(t)\Big)x(t)\,,$$

where f is such that for $s(t) \equiv 0$ we obtain the logistic or Gompertz type equation and α is a bifurcation parameter.

We prove conditions guaranteeing the global stability of the zero steady state. These conditions are necessary and sufficient for a periodic or quasi-periodic suppression rate. Moreover, if the external influence is periodic and the zero steady state is repulsive, the existence of a periodic solution is shown. From the applications point of view, if this equation is used as the description of underlying tumour dynamics and the external influence reflects the tumour treatment, then the global stability of the zero steady state can be interpreted as the cure.

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